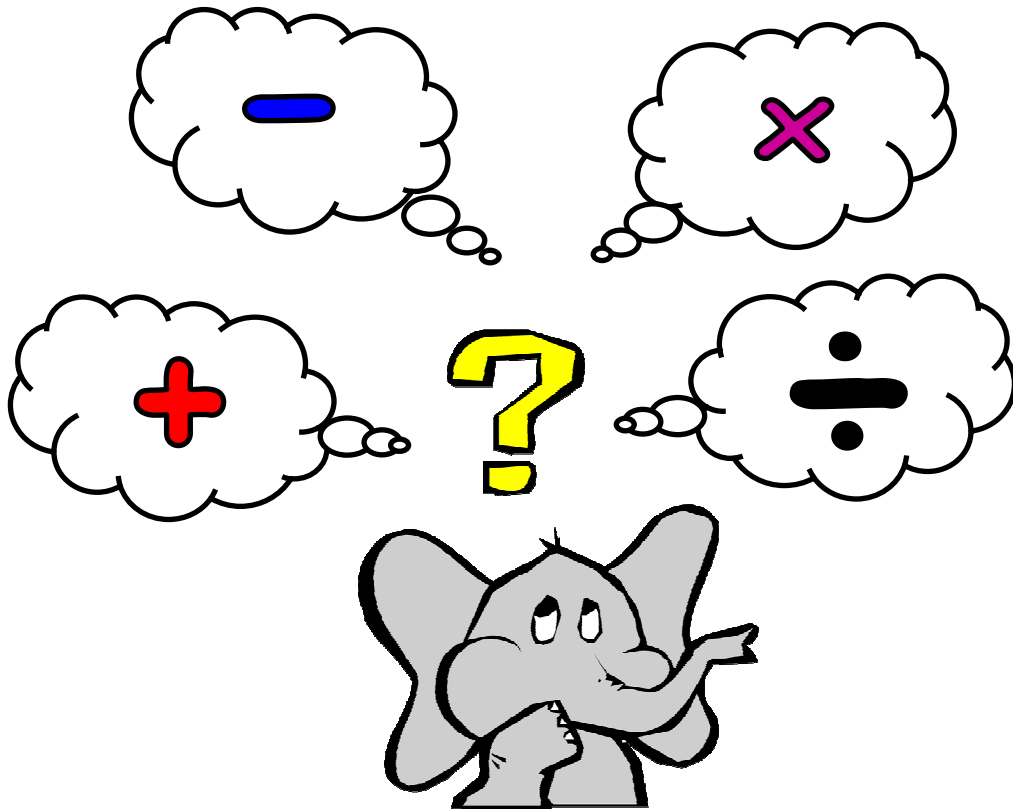


# Progression in Calculations



September 2011

## Introduction

Written methods of calculations are based on mental strategies. Each of the four operations builds on mental skills which provide the foundation for jottings and informal written methods of recording. Skills need to be taught, practised and reviewed constantly. These skills lead on to more formal written methods of calculation.

Strategies for calculation need to be supported by familiar models and images to reinforce understanding. When teaching a new strategy it is important to start with numbers that the child can easily manipulate so that they can understand the concept.

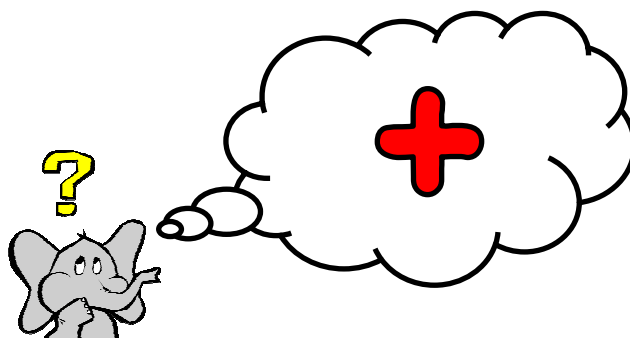
The transition between stages should not be hurried as not all children will be ready to move on to the next stage at the same time, therefore the progression in this document is outlined in stages. Previous stages may need to be revisited to consolidate understanding when introducing a new strategy.

A sound understanding of the number system is essential for children to carry out calculations efficiently and accurately.

## Progression in Teaching Addition

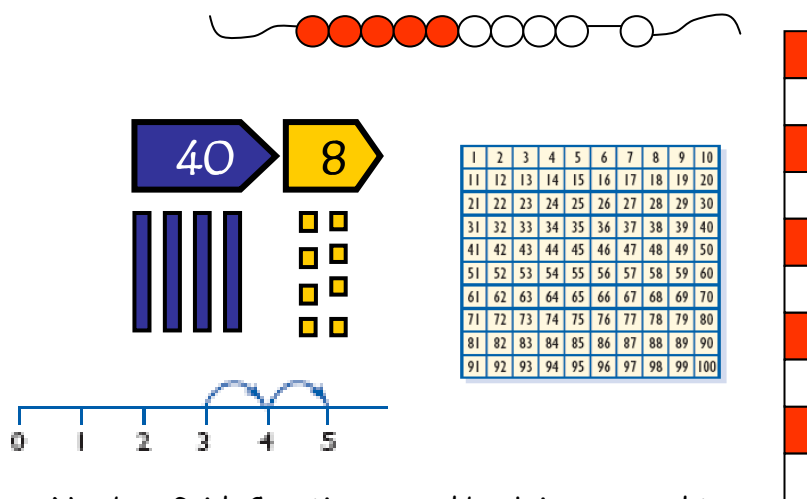
### Mental Skills

Recognise the size and position of numbers  
 Count on in ones and tens  
 Know number bonds to 10 and 20  
 Add multiples of 10 to any number  
 Partition and recombine numbers  
 Bridge through 10



### Models and Images

Counting apparatus  
 Place value apparatus  
 Place value cards  
 Number tracks  
 Numbered number lines  
 Marked but unnumbered number lines  
 Empty number lines  
 Hundred square  
 Counting stick  
 Bead string  
 Models and Images charts  
 ITPs - Number Facts, Ordering Numbers, Number Grid, Counting on and back in ones and tens



### Key Vocabulary

add  
 addition  
 plus  
 and  
 count on  
 more  
 sum  
 total  
 altogether  
 Increase  
 More than

**add and count on**  
**addition plus**  
**more sum total**  
**altogether increase**

Recognise numbers 0 to 10

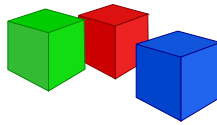
0 1 2 3 4 5 6 7 8 9 10



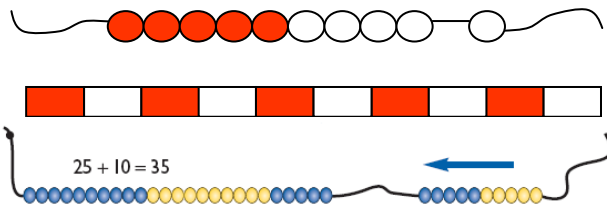
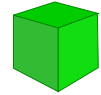
1, 2, 3, 4, 5, 6  
... there are 6  
teddies

Count reliably up to 10 everyday objects

Find one more than a number



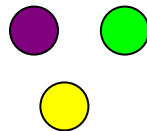
One more than  
three is four



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Count in ones and tens

Begin to relate addition to  
combining two groups of objects



and  makes 5

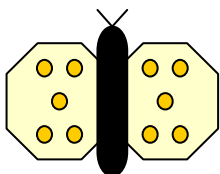
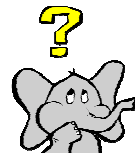
$$3 + 2 = 5$$



Count along a number line to  
add numbers together

Begin to use the + and = signs to record  
mental calculations in a number sentence

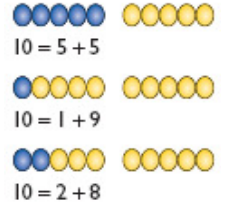
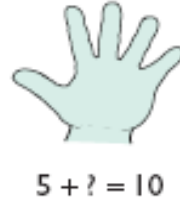
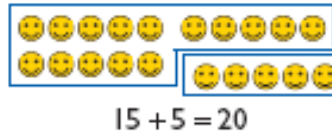
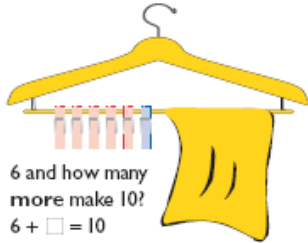
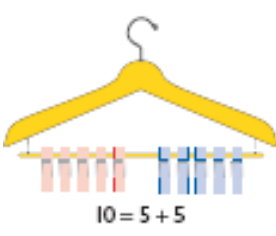
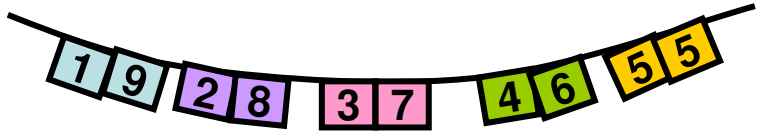
$$6 + 4 = 10$$



$$5 + 5 = 10$$

Know doubles of numbers

Know by heart all pairs of numbers with a total of 10 and 20



$1 + 2 = 3$



$2 + 1 = 3$



$2 + 5 = 7$

2 count on 5



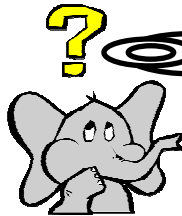
$5 + 2 = 7$



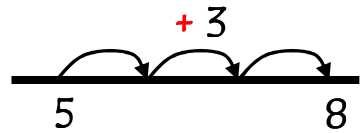
5 count on 2

Know that addition can be done in any order

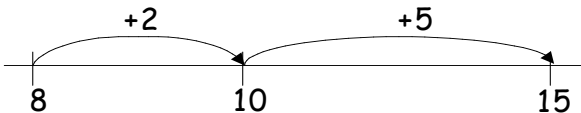
Put the biggest number first and count on



$3 + 5$

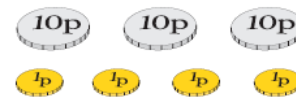
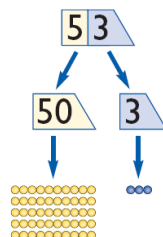


$8 + 7 = 15$



Add two single-digit numbers that bridge 10

Begin to partition numbers in order to add

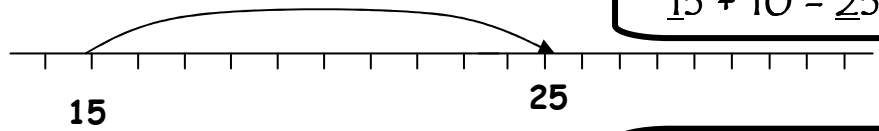


$30p + 4p = 34p$

Know which digit changes when adding 1s or 10s to any number



$$\underline{1}5 + 1 = \underline{1}6$$



$$\underline{1}5 + 10 = \underline{2}5$$



$$\underline{1}5 + 20 = \underline{3}5$$

Adding two two-digit numbers  
(bridging through tens boundary)

Using a number line

OR

Using place value cards and place value apparatus to partition numbers and recombine

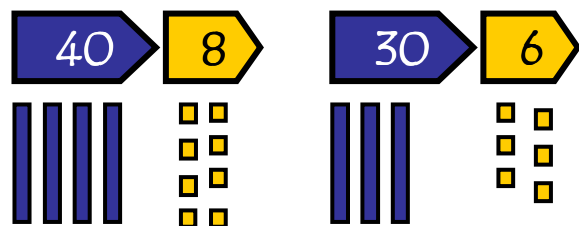
$$48 + 36 = 84$$

$$40 + 30 + 8 + 6$$

$$40 + 30 = 70$$

$$8 + 6 = 14$$

$$70 + 14 = 84$$

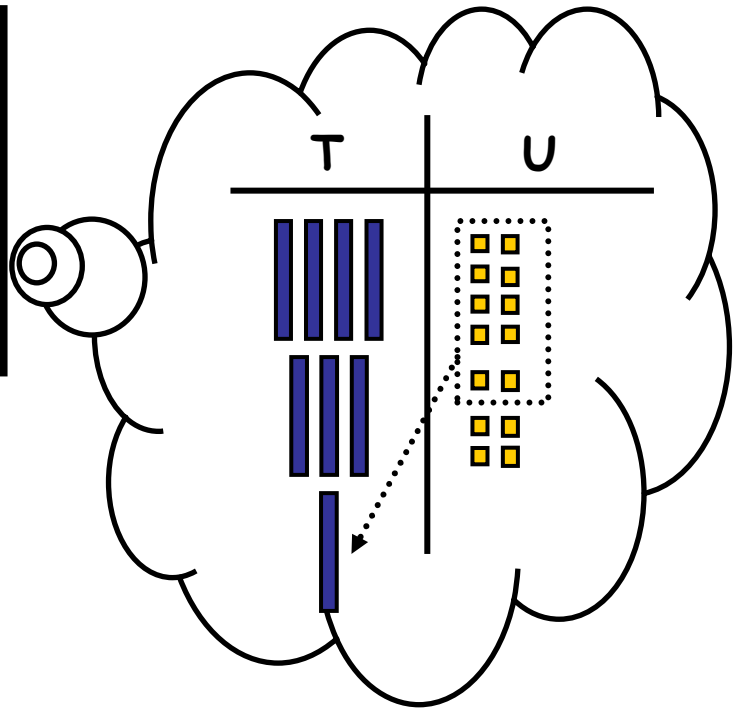


### Expanded method

It is important that the children have a good understanding of place value and partitioning using concrete resources and visual images to support calculations. The expanded method enables children to see what happens to numbers in the standard written method.

$$48 + 36$$

$$\begin{array}{r} 48 \\ + 36 \\ \hline \end{array}$$



$$\begin{array}{r} 48 \\ + 36 \\ \hline 84 \\ \quad 4 \end{array}$$

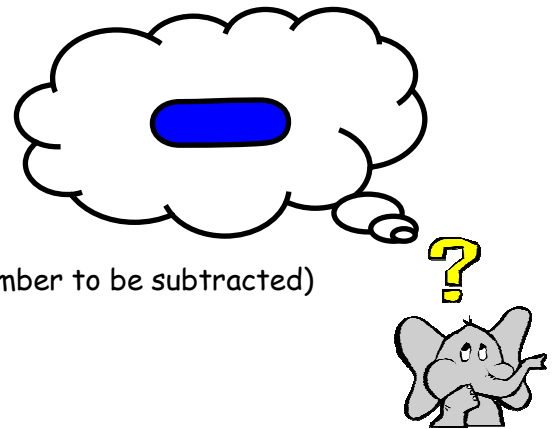
### Standard written method

The previous stages reinforce what happens to the numbers when they are added together using more formal written methods.

## Progression in Teaching Subtraction

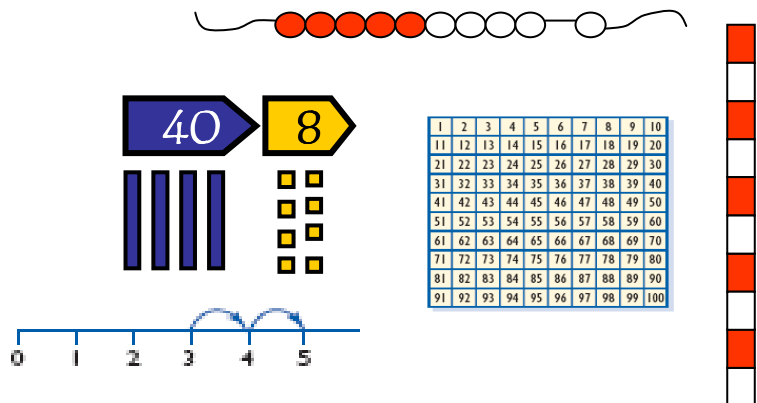
### Mental Skills

Recognise the size and position of numbers  
Count back in ones and tens  
Know number facts for all numbers to 20  
Subtract multiples of 10 from any number  
Partition and recombine numbers (only partition the number to be subtracted)  
Bridge through 10



### Models and Images

Counting apparatus  
Place value apparatus  
Place value cards  
Number tracks  
Numbered number lines  
Marked but unnumbered lines  
Hundred square  
Empty number lines.  
Counting stick  
Bead strings  
Models and Images Charts  
ITPs - Number Facts, Counting on and back in ones and tens, Difference



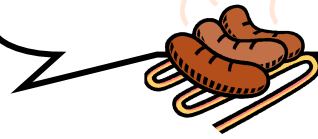
### Key Vocabulary

subtract  
take away  
minus  
count back  
less  
fewer  
difference between

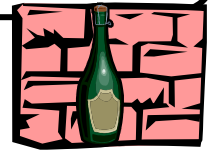
count back take away  
fewer subtract  
minus less  
difference between

Begin to count backwards in familiar contexts such as number rhymes or stories

Five fat sausages frying in a pan ...



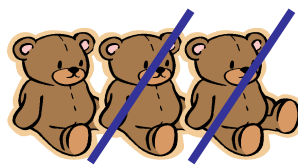
Ten green bottles hanging on the wall ...



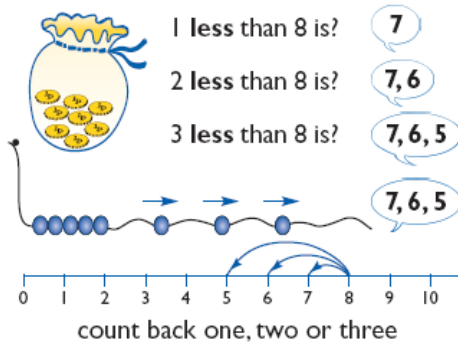
10, 9, 8, 7, ...

Continue the count back in ones from any given number

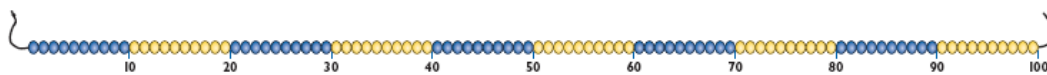
Begin to relate subtraction to 'taking away'



Three teddies take away two teddies leaves one teddy

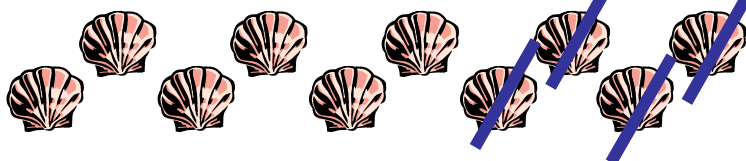


Find one less than a number



Count back in tens

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100



If I take away four shells there are six left



Count backwards along a number line to 'take away'

Begin to use the - and = signs to record mental calculations in a number sentence

Maria had six sweets and she ate four. How many did she have left?



$$6 - 4 = 2$$



$$6 + ? = 10$$

$$10 - 6 = ?$$



$$? + 6 = 10$$

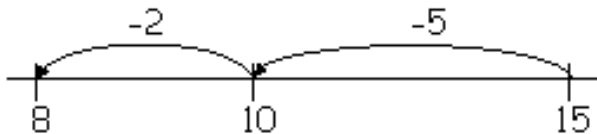
$$10 - 4 = 6$$

		$20 = 12 + 8$	$8 + 12 = 20$
		$20 - 8 = 12$	$20 - 12 = 8$

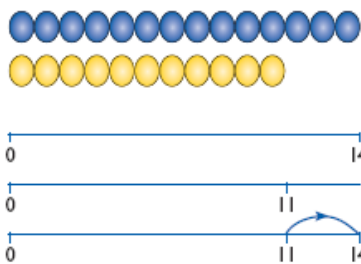
Know by heart subtraction facts for numbers up to 10 and 20

Subtract single digit numbers often bridging through 10

$$15 - 7 = 8$$



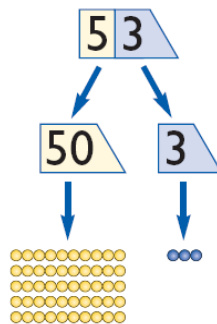
The difference is?



The difference between 11 and 14 is 3.  
 $14 - 11 = 3$   
 $11 + \square = 14$

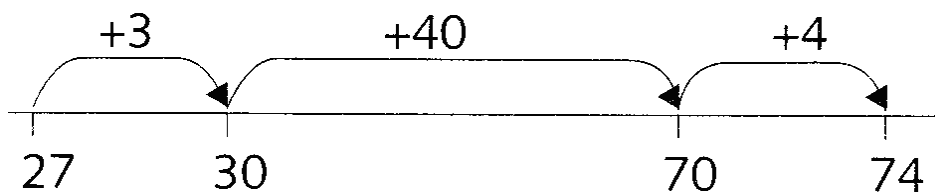
Begin to find the difference by counting up from the smallest number

Begin to partition numbers in order to take away



Subtraction by counting on using a 'washing line' (number line)

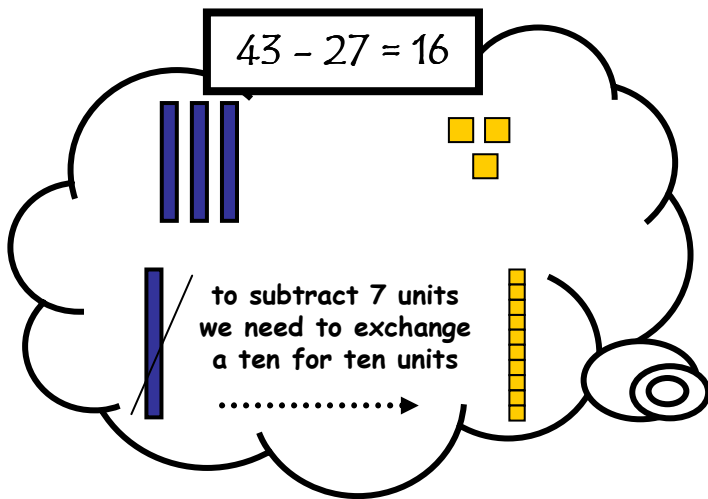
$$74 - 27 = ?$$



Now what's the answer?

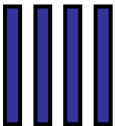

Then just add up the jumps to find the answer!

$$74 - 27 = 47$$



**Expanded method**

It is important that the children have a good understanding of place value and partitioning using concrete resources and visual images to support calculations. The expanded method enables children to see what happens to numbers in the standard written method.

T	U
	
- 2	7

**Standard written method**

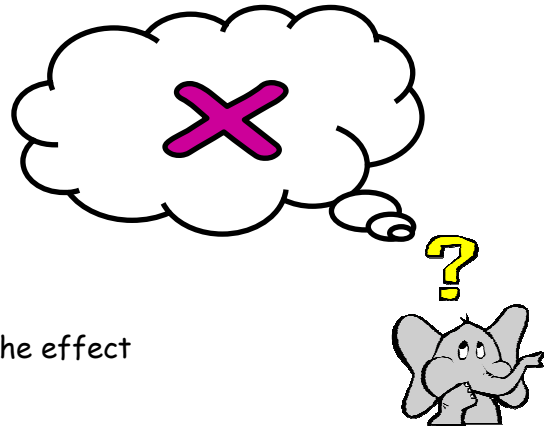
The previous stages reinforce what happens to numbers when they are subtracted using more formal written methods. It is important that the children have a good understanding of place value and partitioning.

$$\begin{array}{r}
 \overset{3}{\cancel{4}}3 \\
 - 27 \\
 \hline
 16
 \end{array}$$

## Progression in Teaching Multiplication

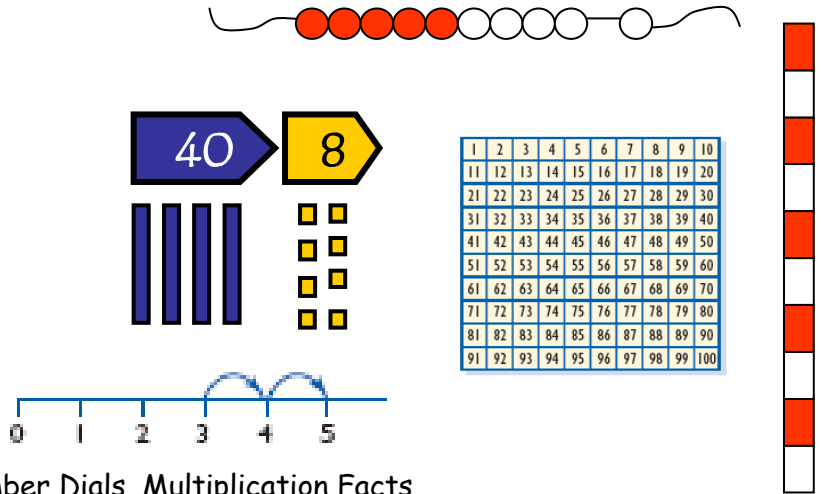
### Mental Skills

- Recognise the size and position of numbers
- Count on in different steps 2s, 5s, 10s
- Double numbers up to 10
- Recognise multiplication as repeated addition
- Quick recall of multiplication facts
- Use known facts to derive associated facts
- Multiplying by 10, 100, 1000 and understanding the effect
- Multiplying by multiples of 10



### Models and Images

- Counting apparatus
- Place value apparatus
- Arrays
- 100 squares
- Number tracks
- Numbered number lines
- Marked but unnumbered lines
- Empty number lines.
- Multiplication squares
- Counting stick
- Bead strings
- Models and Images charts
- ITPs - Multiplication grid, Number Dials, Multiplication Facts

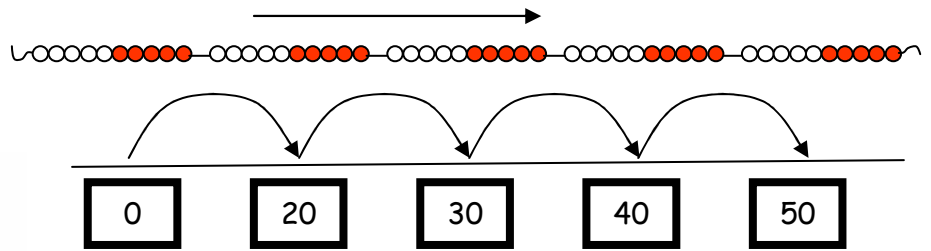


### Vocabulary

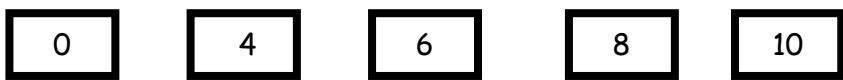
- lots of
- groups of
- times
- multiply
- multiplication
- multiple
- product
- once, twice, three times
- array, row, column
- double
- repeated addition

**multiplication**      **product**  
**once, twice, three times**  
**double**      **groups of**  
**repeated addition**      **lots of**  
**array, row, column**      **multiply**  
**times**      **multiple**

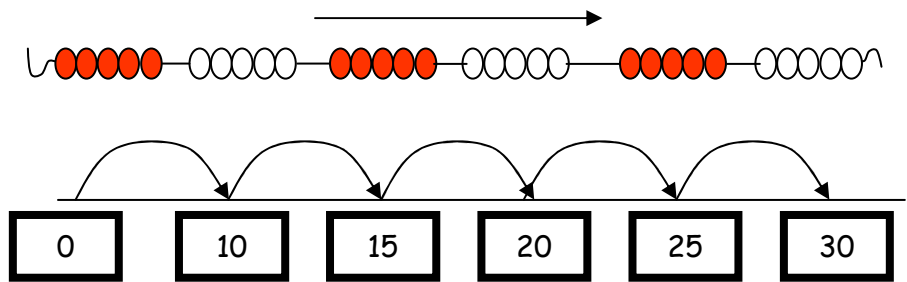
Count in tens from zero



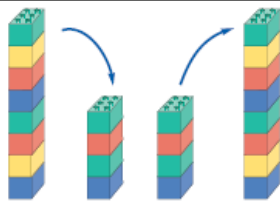
Count in twos from zero



Count in fives from zero



Know doubles and corresponding halves



half of 8 is 4  
 $8 \div 2 = 4$

double 4 is 8  
 $4 \times 2 = 8$

Know multiplication tables to  $10 \times 10$

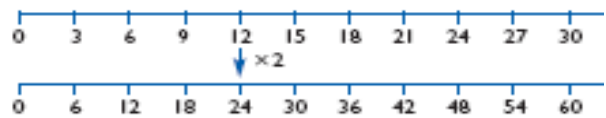
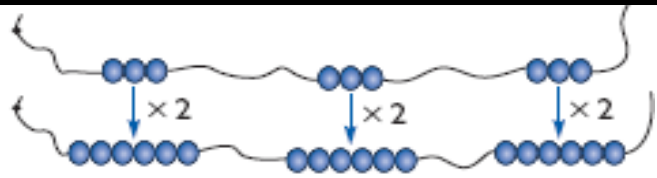
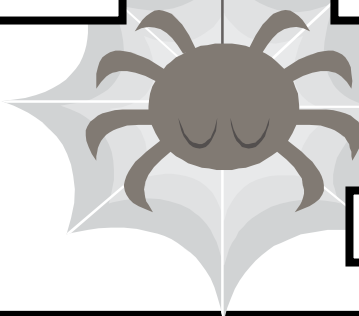
$2 \times 5 = 10$

$\times 5$

$6 \times 5 = 30$

$3 \times 5 = 15$

$8 \times 5 = 40$



$12 \times 2 = 24$

Twice as many

Use known facts to work out new ones

Understand that ...

$$24 \times 20 = 24 \times 2 \times 10$$

$$24 \times 50 = 24 \times 5 \times 10$$

Use factors to multiply



$$2 + 2 + 2 + 2$$

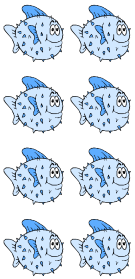
Understand multiplication  
as repeated addition

$$2 + 2 + 2 + 2 = 8$$

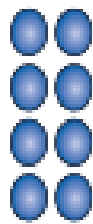
$$4 \times 2 = 10$$

2 multiplied by 4

4 lots of 2

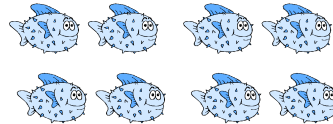


$$2 \times 4$$

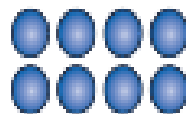


$$2 \times 4 = 8$$

$$4 \times 2 = 8$$



$$4 \times 2$$

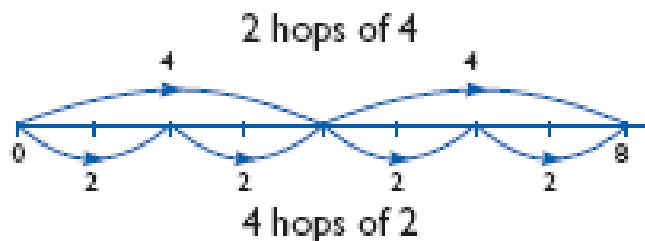


$$4 \times 2 = 8$$

$$2 \times 4 = 8$$

Understand  
multiplication  
as an array

Understand how to  
represent arrays  
on a number line



$$4 \times 13$$

Use place value apparatus to support the multiplication of U x TU alongside the grid method

	x	10	3
4		40	12

$$40 + 12 = 52$$

Multiplying TU x TU

$$14 \times 33$$

		30	3
10		300	30
4		120	12

$$= 330 +$$

$$= \begin{array}{r} 132 \\ \hline 462 \end{array}$$

300
120
30
+ 12
<hr/>
462

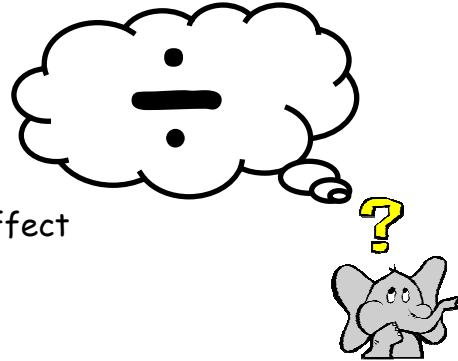
$$\begin{array}{r} 56 \\ \times 27 \\ \hline 42 \quad (7 \times 6) \\ 350 \quad (7 \times 50) \\ 120 \quad (20 \times 6) \\ 1000 \quad (20 \times 50) \\ \hline 1512 \end{array}$$

$$\begin{array}{r} 56 \\ \times 27 \\ \hline 392 \\ 1120 \\ \hline 1512 \end{array}$$

## Progression in Teaching Division

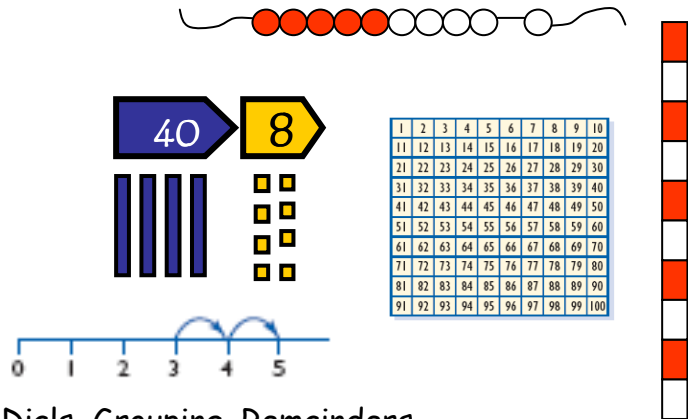
### Mental Skills

- Recognise the size and position of numbers
- Count back in different steps 2s, 5s, 10s
- Halve numbers to 20
- Recognise division as repeated subtraction
- Quick recall of division facts
- Use known facts to derive associated facts
- Divide by 10, 100, 1000 and understanding the effect
- Divide by multiples of 10



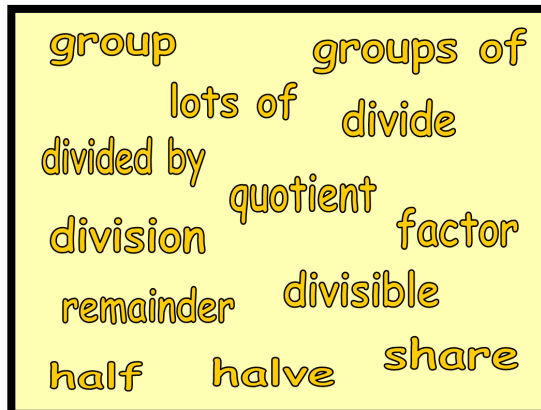
### Models and Images

- Counting apparatus
- Arrays
- 100 squares
- Number tracks
- Numbered number lines
- Marked but unnumbered lines
- Empty number lines.
- Multiplication squares
- Models and Images charts
- ITPs - Multiplication grid, Number Dials, Grouping, Remainders

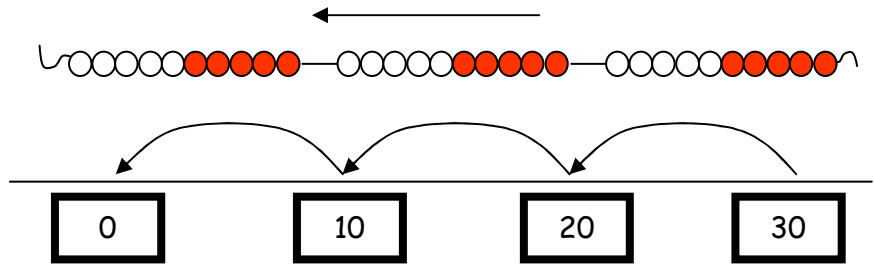


### Vocabulary

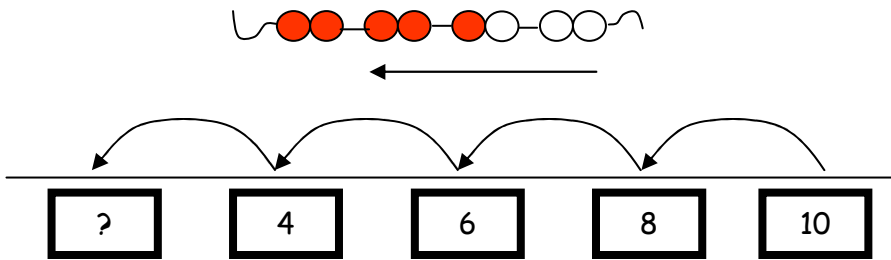
- lots of
- groups of
- share
- group
- halve
- half
- divide
- division
- divided by
- remainder
- factor
- quotient
- divisible



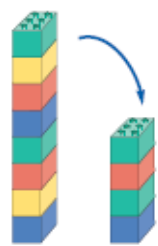
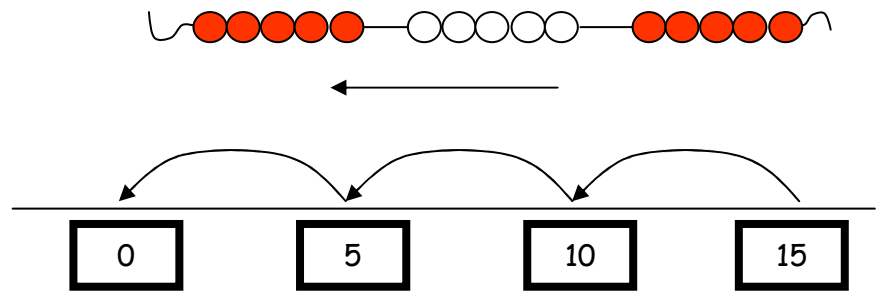
Count back in tens



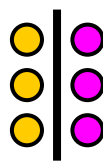
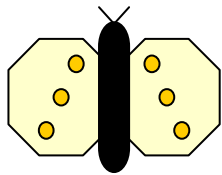
Count back in twos



Count back in fives



half of 8 is 4  
 $8 \div 2 = 4$



Half of 6 is 3

$\frac{1}{2}$  of 6 = 3

Know halves

Use known multiplication facts to work out corresponding division facts

$$\text{If } 2 \times 10 = 20$$

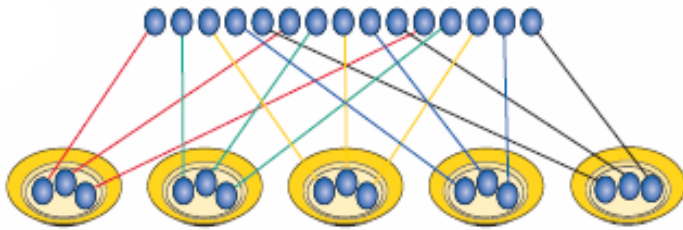
then

$$20 \div 10 = 2$$

$$20 \div 2 = 10$$

$$15 \div 5 = 3$$

15 shared between 5



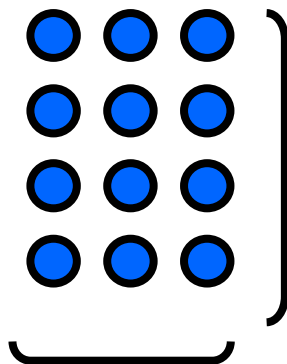
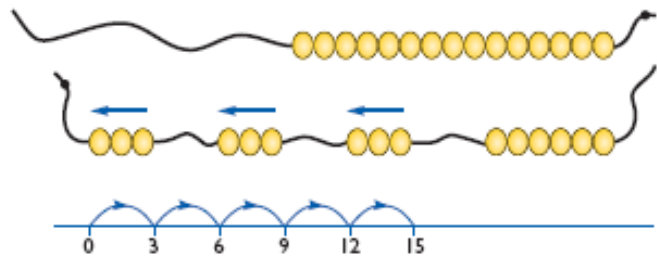
Understand division  
as sharing

Understand division  
as grouping

How many 3s  
in 15?



$$15 \div 3 = 5$$



12 divided into groups  
of 3 gives 4 groups

$$12 \div 3 = 4$$

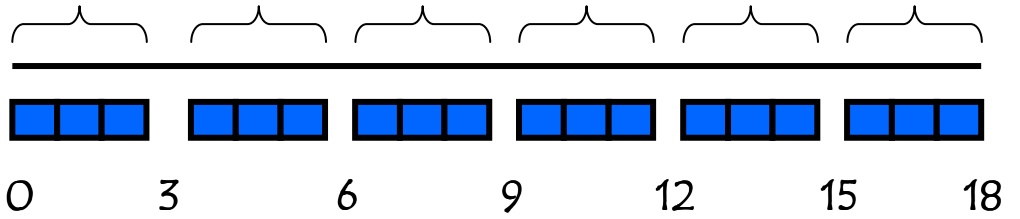
12 divided into groups  
of 4 gives 3 groups

$$12 \div 4 = 3$$

Reinforce division as  
grouping through the  
use of arrays

Represent 'groups' for division on a number line using apparatus alongside the line

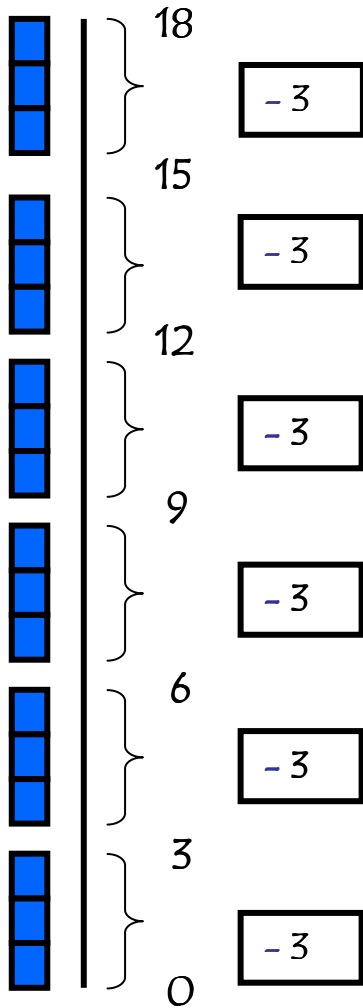
18 divided into groups of 3  
 $18 \div 3 = 6$



$18 \div 3 = 6$



$18 \div 6 = 3$

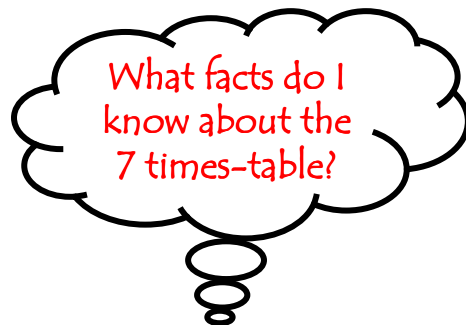


$18 \div 3 = 6$

$$\begin{array}{r} 18 \\ - 3 (1 \times 3) \\ \hline 15 \\ - 3 (1 \times 3) \\ \hline 12 \\ - 3 (1 \times 3) \\ \hline 9 \\ - 3 (1 \times 3) \\ \hline 6 \\ - 3 (1 \times 3) \\ \hline 3 \\ - 3 (1 \times 3) \\ \hline 0 \end{array}$$

Understand division as repeated subtraction using a vertical line and apparatus to make the links

Children need to see that as the numbers get larger, large chunk subtraction is the more efficient method. Multiples of the divisor (large chunks) are taken away. Multiplication facts are needed to see the size of the 'chunk'.



$$100 \div 7 = 14 \text{ r } 2$$

$$\begin{array}{r} 100 \\ - 70 \quad (\underline{10} \times 7) \\ \hline 30 \\ - 28 \quad (\underline{4} \times 7) \\ \hline 2 \end{array}$$

$$518 \div 7 = 74$$

$$\begin{array}{r} 518 \\ - 350 \quad (\underline{50} \times 7) \\ \hline 168 \\ - 140 \quad (\underline{20} \times 7) \\ \hline 28 \\ - 28 \quad (\underline{4} \times 7) \\ \hline 0 \end{array}$$

Fact Box	
1	$\times 7 = 7$
2	$\times 7 = 14$
5	$\times 7 = 35$
10	$\times 7 = 70$
20	$\times 7 = 140$
50	$\times 7 = 350$
100	$\times 7 = 700$

$$\begin{array}{r} 46 \\ 8 \overline{) 33648} \\ \hline 144 \text{ r } 3 \end{array}$$

$$5 \overline{) 72223}$$

$$\begin{array}{r} 144.8 \\ 5 \overline{) 72224.40} \end{array}$$

Encouraged to find approximate answers first by rounding up or down

2 ways of expressing remainders